Effect of flexible pin on the dynamic behaviors of wind turbine planetary gear drives

CaiChao Zhu¹, XiangYang Xu¹, Teik Chin Lim², XueSong Du¹ and MingYong Liu¹

Abstract
Flexible pins eliminate the need for straddle mounting, and therefore enable the maximum possible number of planets to be used for any particular epicyclic ratio of power transmission systems. Having more planet gears will significantly increase the input torque density. In this type of design, the pin stiffness and position tolerances are important parameters as they affect the dynamic performances significantly. The present study addresses this issue by modeling, the design of double cantilevered flexible pin, and analyzing the contributions of pin stiffness and misalignment applying the lumped parameter approach. The proposed model formulates the coupled lateral-torsional dynamic response of a planetary spur gear, including the effects of mesh stiffness and phasing as a function of pin error. The resultant equations of motion are applied to examine the effects of pin stiffness and position errors on the natural modes and structural dynamic response. The effects of pin stiffness on deviation of the tooth contact forces of the sun-planet and ring-planet gear pairs are analyzed to understand the relationship between mesh characteristic and input speed variations. The calculated supporting forces of the planet gear are examined to understand the load sharing characteristic due to pin errors, pin stiffness and input load of the power transmission system.

Keywords
Planetary gear, flexible pin, mesh stiffness, gear misalignment error, dynamic mesh force, load sharing

Introduction
Planetary gears offer several major advantages over parallel axis gears, including the ability to deliver a wide range of speed and torque ratios by varying the input, output, and reaction members, and its inherently compact design and greater torque-to-weight ratio due to existence of multiple parallel paths. These advantages led to extensive use of planetary gears in multi-speed, high power density applications such as wind turbines, rotorcrafts and tunnel boring transmissions. Despite their already compact nature, there are continual demands for achieving still greater power density that can consequently lead to novel designs and new applications. The design of a flexible pin is a possible method to achieve greater performance. This is because the use of a flexible pin eliminates the need for straddle mounting, and therefore enables the maximum possible number of planets to be used for any particular epicyclic ratio. This type of design can offer greater torque density by simply increasing the number of planets. In addition, this concept can be helpful in improving load sharing where if one planet has the tendency to take more load than the others, the extra flexibility will allow the necessary deflections to ensure that each planet gear takes its equal share of load. Because of this potential advantage, the design has led to an increased interest in understanding the load sharing characteristics with four or more planets. To address this concern, the focus of this article is to analyze the influence of a flexible pin on planetary gear dynamics,
and to propose an approach that can accurately evaluate the effect of a flexible pin early in the design stage.

Pin stiffness and tolerance are two important design variables that can have an impact on the dynamic performance of the planetary gear, as alluded to in the above discussion. The advantage is that both pin stiffness and bearing stiffness are parameters that can be modeled and designed relatively easily without the need to consider gear parameters directly. Some earlier studies have examined the effects of planet gear parameters on dynamic behavior, and have led to some well-known results. Lin and Parker\cite{1,2} have performed some significant analytical modeling work on the natural frequencies and vibration modes of planetary systems, and their papers computed the sensitivity of natural frequencies and vibration modes to bearing stiffness and mesh stiffness parameters. In another study, Lin and Parker\cite{3} use an analytical approach to study the natural frequency veering phenomenon in planetary gear systems. This piece of work analytically characterizes the behavior of eigenvalue veering caused by key design parameters in planetary gears. However, these models did not address the effect of flexible pin stiffness and pin errors on gear dynamics and the behavior of load sharing was not explicitly mentioned. Kahraman\cite{4} constructed a mathematical model to examine the dynamics of a planetary gear stage, which can be set to any arbitrary number of planets, and has the ability to accommodate many possible gear sizes and tolerance variations for both fixed and floating sun gears. In addition, Kahraman presented a limited parametric study of several of those variables on one specific design of a gearbox. In another study published in 1999 by Kahraman,\cite{5} the load sharing of planetary gear sets is studied using a mathematical model and also experimental setup, and his results showed reasonably good agreement between experimental results and mathematical predictions for a four-planet system. In recent study, Montestruc\cite{6} used a finite element method to computer static pin structural stress of three type flex pins under the same load, also compare the difference between variations of Hicks flexible cantilever pins and straddle type carrier on load sharing and results showed that the low spring constant flexible pins have significantly superior load sharing characteristics. However, in that study, the planet bearing stiffness is not considered as a parametric variable, even though it is included in the model. Ligata et al. in a 2008 paper,\cite{7} extended Kahraman’s effort\cite{8} to include the numbers of planets and torque as parametric variables in their experiments, and again attained good agreement with the proposed theory.

It is general knowledge that in practical planetary gear systems, some amount of manufacturing errors, such as carrier, sun or other element position errors, are unavoidable, and they can induce undesirable unequal load sharing.\cite{8-10} Singh\cite{11,12} made outstanding contributions to understand load sharing. He gives a static physical explanation for unequal load sharing due to manufacturing errors. And his load sharing predictions and the equivalent error formulation are also validated by comparing with the results from an experimentally validated computational model. However, in order to minimize the effect from these errors, it is also essential that we have a deep understanding of their effect on dynamic behaviors and be able to develop an effective countermeasure at the design stage to mitigate these undesirable effects. In many industrial applications, especially wind turbine transmission systems, reliable models that can provide an accurate prediction of this imperfection of load sharing has been lacking, even though much studies have been completed in the past. In one study, Kahraman and Vijayakar\cite{8} developed a finite element model (FEM) to study the effects of three different groups of gear manufacturing errors on the load sharing for a planetary gear set with small modulus and light load. Hidaka and Terauchi\cite{13} also examined the effects of the number of planet gears on load sharing with a floating sun or carrier. Singh\cite{14} indicated that more planets can increase the load density but at the same time load sharing with more than three planets is not perfect. However, the studies by Hidaka and Terauchi,\cite{13} and Singh\cite{14} focused on the influence of the number of planet gears on load sharing, and concluded that at least one central member must be allowed to float to allow for radial redistribution of the total load amongst the planets. It may be noted that in spite of recent planetary gear work, there are surprisingly very few studies on the dynamics of planetary gears employed in wind turbine transmission systems and understanding of the effect of flexible pin. This present study attempts to address this gap by computing the effects of pin stiffness and pin position tolerances on load sharing, and target the studies mainly on wind turbine power transmissions.

In addition, it is widely known that the sun-planet (S-P) and ring-planet (R-P) gear pair mesh stiffness functions are key factors that must be accounted for in the dynamic analysis of planetary systems. In fact, many prior studies\cite{15-18} have been carried out to calculate the mesh stiffness of a gear pair. FEMs are the most popular tool used.\cite{15,16} In a number of cases, analytical methods also showed reasonable results in predicting tooth mesh stiffness of a gear pair.\cite{17,18} Those analytical solutions that correlate well with FEM results can reduce computation time significantly. In one specific study, Amburisha and Parker\cite{19} compared the accuracy of using a lumped parameter model to that of the finite element model when examining the dynamic behavior of spur-type planetary gears. Their calculations verified...
the effectiveness of the lumped-parameter model containing both translational and rotational coordinates in simulating the dynamics of planetary gears. Again, mesh stiffness and phasing calculations with effect of pin error included are not analyzed in all of the above studies. In the present study, a coupled lateral-torsional lumped parameter dynamic model of a wind turbine planetary spur gear is proposed. Also, the mesh stiffness and mesh phasing models with pin error are developed analytically and incorporated into the proposed lumped parameter model for use to analyze gear dynamical behaviors.

**Dynamic modeling**

A generic precision planetary gear set used in wind turbine application is shown in Figure 1. This transmission will be employed as the basis for the present study. The carrier \( c \) is the input member, the floating sun gear \( s \) is the output member, and the ring gear \( r \) is the reaction member. The design parameters of the system are shown in Table 1. Planetary gear systems have typically been equipped with straddle mounted planet gear idlers with pins supported on the carrier. However, with the flexible pin design of cantilever type carrier, the need for straddle mounting can be avoided but raise the bending stresses in the pin. Fox and Jallat\(^{20,21} \) design a new integrated flexible pin to decrease these bending stresses and also can keep a low stiffness. This type of ‘floating’ flexible pins are of advantage in low speed, heavy load transmissions, such as in wind turbines. The structural design of the flexible planet pins studied in this article is shown in Figure 2(a) and (b).

The pair of forces acting in the middle when measured along the tooth width direction of the planet gear shaft can be calculated as
\[
f = \frac{2T}{nD}
\]
where \( T \), \( n \) and \( D \) are the total torque transmitted through the carrier, the number of planet gears, and the pitch circle diameter of the planet gears, respectively. Bending moments of the two ends of planet gear shaft can be calculated from
\[
M_1 = f \times \left( \frac{l}{2} \right), \quad M_2 = f \times \left( \frac{l}{2} + \frac{l}{2} \right) - M_1,
\]
and hence, \( M_1 = M_2 \). Therefore, the bending moment at the center point becomes zero and the pins may be considered as two serially connected cantilevered beams. The slope at each end is zero. And the moment loads on the gear act to correct the force vector position back to the center of the gear width. The central planet pin thus deflects in an elongated ‘S’ shape due to the pair of forces of acting on the planet gear shaft. This ensures that the planet gear shaft remains horizontal under deformation and bearing force acting on the planet gear shaft is perpendicular to the shaft axis line. This also means that the planetary shaft is effectively rigid similar to a bearing.

Since only spur gears are assumed in this design, all components can be defined sufficiently with 3 degrees of freedom, that are one rotational coordinate \( u \) and two translational coordinates \( x, y \). The geometric relationship between meshing components, sun-planet (S-P) meshing and ring-planet (R-P) meshing, are illustrated in Figure 3(a). The relative position between planet gear and carrier is shown schematically in Figure 3(b).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sun</th>
<th>Planet</th>
<th>Carrier</th>
<th>Ring</th>
</tr>
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<tr>
<td>Number of teeth, ( Z )</td>
<td>21</td>
<td>37</td>
<td>–</td>
<td>95</td>
</tr>
<tr>
<td>Modulus, ( m ) (mm)</td>
<td>15</td>
<td>15</td>
<td>–</td>
<td>15</td>
</tr>
<tr>
<td>Tooth width (mm)</td>
<td>335</td>
<td>315</td>
<td>1596</td>
<td>335</td>
</tr>
<tr>
<td>Mass, ( m ) (kg)</td>
<td>205</td>
<td>289</td>
<td>1596</td>
<td>1538</td>
</tr>
<tr>
<td>( li^2 ) (kg)</td>
<td>199</td>
<td>271</td>
<td>1681</td>
<td>1847</td>
</tr>
<tr>
<td>Base circle diameter (mm)</td>
<td>286</td>
<td>503</td>
<td>435</td>
<td>1292</td>
</tr>
<tr>
<td>Pressure angle (°)</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 1.** Planetary gear train of wind turbine (keys: \( s \) – sun gear, \( r \) – ring, \( c \) – carrier, and \( p \) – planetary gear, \( 1, 2, 3, 4 \) – serial number of the planet).

**Figure 2.** Flexible planet pin (labels: 1 – carrier, 2 – central planet pin, 3 – planetary shaft). (a) Mounted pin, (b) Free-body diagram.
The coordinates is defined as follows: the ring, carrier and sun gear translational displacements \( x_i, y_i, z_i \), \( i = r, c, s \), and planet translational displacement \( t_n, r_n, n = 1, 2, 3, 4 \) are measured with respect to a rotating frame fixed to the carrier with origin at \( O_c \). The displacements \( x_i, y_i, z_i \) are directed towards the equilibrium position of planet 1, and \( t_n, r_n \) are the tangential and radial deflections of the \( n \)-th planet, and \( u_j, j = r, c, s, n \) are angular displacements of ring, carrier, sun and planet gears, respectively. The symbol \( \Delta \) denotes the compression of elastic elements, and the compression of gear mesh is defined as

\[
\text{S-P mesh}: \quad \Delta_{sn} = y_s \cos \psi_{sn} - x_s \sin \psi_{sn} - t_n \sin \alpha_{sn} - r_n \cos \alpha_{sn} + u_1 + u_n + e_s(t)
\]

\[
\text{R-P mesh}: \quad \Delta_{rn} = y_r \cos \psi_{rn} - x_r \sin \psi_{rn} + t_n \sin \alpha_{rn} - r_n \cos \alpha_{rn} + u_1 - u_n + e_r(t)
\]

where the angles \( \alpha_{sn}, \alpha_{rn} \) are the pressure angle between S-P and R-P, respectively, \( \psi_{sn} = \psi_n - \alpha_{sn}, \psi_{rn} = \psi_n + \alpha_{rn} \), and functions \( e_s(t), e_r(t) \) are the transmission errors of the S-P and R-P meshes, respectively. Also, the compression of the supports is defined as

\[
\text{Planet radial support}: \quad \Delta_{nr} = y_c \sin \psi_n + x_c \cos \psi_n - t_n
\]

\[
\text{Planet tangential support}: \quad \Delta_{nt} = y_c \cos \psi_n - x_c \sin \psi_n - r_n + u_c
\]

According to the actual condition of planetary gear train, the ring is fixed but this is simulated by very high support stiffnesses, and the carrier and sun are respectively regarded as the input and output components. The sun gear, ring gear and planet gear mesh model is shown in Figure 4. Input angular speed \( \omega_c \) is speed of carrier, and \( k_{sn}(t), k_{rn}(t) \) are mesh stiffness functions of S-P and R-P mesh, respectively. The symbols \( k_s, k_r, k_c \) are bearing support stiffnesses of sun gear, ring gear and carrier; \( k_{sn}, k_{rn}, k_{sc} \) are circumferential stiffnesses of sun gear, ring gear and carrier; \( c_{sn} \) and \( c_{rn} \) are mesh damping elements that are assumed to be the same.

**Figure 3.** Kinematic relationships for deriving component deflections of the planetary gear train.

**Figure 4.** Sun gear, ring gear and planet gear mesh model.
for each S-P and R-P mesh; \( c_r, c_r, c_c \) are bearing damping elements of sun gear, ring gear and carrier; \( c_{ur}, c_{ur}, c_{uc} \) are circumferential damping elements of sun gear, ring gear and carrier; and \( k_{pb} \) and \( c_{pb} \) are total supporting stiffness and damping elements of planet bearing and flexible pin. The differential equations of motion for the sun gear, ring gear, carrier and planet gears are abbreviated, which is similar to Lin and Parker’s model.1,2 Their model has been applied to do some dynamic analysis. Here, the authors improved Lin and Parker’s model to investigate the flexible pin effect. In this improved model, the modeling assumptions are similar to those in Lin and Parker’s model, but floating pins and floating sun gear are simulated by the lower pin stiffness and the very low sun supporting stiffness which are different from Lin and Parker’s model. The damping and error excitation are added in this model, while these are not considered in Lin and Parker’s model. Dynamic mesh forces and planet gear supporting forces will be determined based on this improved model. The more compactly global assembling dynamical equation of the 3n + 9 degrees of freedom can be obtained in matrix form as follows

\[
M \ddot{X} + (C + \omega_c G) \dot{X} + (K_h + K_m(t) - \omega_c^2 K_{oa})X = T + F(t) \tag{5a}
\]

\[
X = (x_1, y_1, u_s, x_r, y_r, u_r, x_c, y_c, u_c, I_p, r_p, u_b) \tag{5b}
\]

where \( X \) is the system displacement vector, \( M \) is the mass matrix, \( C \) is the damping matrix, \( G \) is the gyroscope matrix, \( K_h \) is the supporting stiffness matrix, \( K_{oa} \) is of centripetal stiffness matrix, \( K_m \) is the time varying mesh stiffness matrix, and \( T \) represents external torque vector. Note that \( G \) and \( K_{oa} \) are due to the high-speed carrier rotation. Also, \( \omega_c \) is the input speed of the wind turbine carrier is limited to the range of 0–19 r/min. Since these speeds are relatively low, the effects of \( G \) and \( K_{oa} \) can be neglected. The exciting force \( F(t) \) is induced by the kinematic transmission error due to tooth profile error, elastic deformation and misalignment. For brevity, the detailed contents of these matrices are not given here.

**Pin parameters**

**Stiffness and position error**

The supporting stiffness of planet gear shaft is due to the flexible pin stiffness \( k_p \) and bearing stiffness \( k_b \); acting in series. The total supporting stiffness \( k_{pb} \) can then be expressed as

\[
k_{pb} = 1/(1/k_p + 1/k_b) \tag{6a}
\]

\[
k_p = \frac{1}{\int C_p} \int M \partial M \frac{\partial x}{\partial f} \text{ d}x \quad \text{d}x \tag{6b}
\]

In practice, as discussed earlier, pin manufacturing and assembly errors often exist, which may significantly affect gear meshing characteristics. Hence, it is desirable to quantify the influences of these errors and identify the critical tolerances that should be more closely controlled. In the present analysis, the positioning errors of flexible pin are specified along the radial \( e_{rad} \) and tangential \( e_{tan} \) directions. Figure 5 illustrates the positioning errors of one of the planet gears. Note that the center \( O_p \) is the idealized location of the center of the pinion pin-hole inside the carrier. Here, the radial direction is defined along the line connecting the sun-pinion centers, with a positive value resulting in an increase in the sun-pinion center distance. The positive tangential direction is defined as the direction that would bring pinion \((i+1)\) closer to the previous pinion \(i\).

**Effect of pin error**

The gear mesh stiffness formulation employed in this study is based on the work by Wu et al.,17 and Chaari et al.18 They applied the potential energy method to derive the effective mesh stiffness. The total potential energy \( U \) in a pair of meshing teeth is the summation of the Hertzian energy term \( U_h \), bending energy term \( U_b \), shear energy term \( U_s \), and compressive energy term \( U_c \), which can be written as

\[
U = U_h + U_{tb} + U_{sx} + U_{ns} + U_{ph} + U_{ts} + U_{la} \tag{7}
\]
where subscript $n$ denotes $n$-th planet gear, and subscript $i$ referring to either the sun gear $s$ and ring gear $r$.

Accordingly, for a pair gear meshing, the total effective mesh stiffness $k_m$ can be expressed as follows

$$k_m = \frac{1}{1/k_{sb} + 1/k_{ri} + 1/k_{bi} + 1/k_{ns} + 1/k_{ia} + 1/k_{ia}}$$

(8)

where $k_{sb}$, $k_{ri}$ denotes bending mesh stiffness of planet, sun and ring gear meshing teeth ($i = s, r$), $k_{ns}$, $k_{ia}$ denotes shear mesh stiffness of planet, sun and ring meshing teeth ($i = s, r$), and $k_{bi}$ denotes mesh stiffness caused by Hertzian contact.

Pin position errors influence the position of the planet gear tooth flanks that mesh with the sun and ring gears. The radial position error can be simulated as center distance error as long as their amplitudes are less than the tooth bottom clearance. However, the tangential errors have effect on gear meshing phasing and stiffness. Therefore, in order to formulate the gear mesh stiffness, radial error and tangential error can be replaced by the center distance error. When the center distance is changed, the effective pitch diameter and pressure angle are also changed. Then, the new contact ratio and mesh stiffness are recalculated. The center distance errors with amplitudes varying from positive 100 $\mu$m to as high as positive 500 $\mu$m have been prescribed on planet gear 1 with all the other pinions assumed to be ideally positioned. Given this case, the predicted gear mesh stiffness variations is plotted in Figure 6. The results here show that increasing center distance errors tend to lower the actual contact ratio on the S-P mesh and increase the R-P mesh one.

In the analysis of planetary gear system, it is critical that the planet gear phasing relationships be accurately accounted for in the analytical models and computer simulations. However, not all published studies apply this proper phasing characteristic. Parker and Lin$^{22}$ studied the phasing relation between the S-P and R-P meshes. Their mesh phasing model provides more realistic representations of the dynamic characteristic of the meshing gears. However, as noted previously, no prior study has been found that examined the effects of pin errors on gear mesh phasing.

Figures 7(a) and (b) show the mesh phasing details between the S-P and R-P meshes, all phasing is expressed as a fraction of the mesh period. Here, the period is the same for the S-P and R-P meshes. Also, only the decimal portion of mesh period need to be considered in this study because the integer number portion represents the number of mesh cycles of phase difference. For the S-P mesh, when planet gear 1 reaches the position of planet gear $n$ from initial position, it would have traversed $\frac{Z_s \psi_n}{2\pi}$ mesh cycle. If it is a counter-clockwise planet rotation, the S-P mesh phasing relations are defined as $\Delta \psi_{sn} = -\text{mod}(Z_s \psi_n/2\pi)$. Similarly, the R-P mesh phasing relations are defined as $\Delta \psi_{rn} = \text{mod}(Z_r \psi_n/2\pi)$, where mod means the remainder calculation. Tangential errors can cause the planet coming into contact earlier (or later) $\Delta \psi_t$ than all the other planets. So, after considering the tangential error, $\Delta \psi_{sn}$, $\Delta \psi_{rn}$ can be easily modified to $\Delta \psi_{sn}$, $\Delta \psi_{rn}$ as shown in Figure 8.

When the center distance error exist, the pressure angles are unequal, and will be modified as $\alpha_1$, $\alpha_2$. To determine what the phase of R-P mesh is when the S-P mesh occurs at its pitch point, the point T in the R-P contact region $B_2E_2$ is defined. Because R-P mesh occurs on the opposite planet gear tooth face than S-P mesh because of simultaneous internal and external mesh, the planet gear base circle tooth thickness $t_b$...
should be subtracted. Consequently, the magnitude of the relative phase between the pitch points is

$$P_2 T = 1 - \text{mod} \left( \frac{B_2 T}{P_b} \right) P_b - \left( R_p \tan \alpha_2 - \sqrt{R_{ra}^2 - R_r^2} \right)$$

where

$$B_2 T = [R_p \tan \alpha_1 + R_p(\pi - \alpha_1 - \alpha_2) + R_p \tan \alpha_2]$$

$$- R_s \tan \alpha_2 + \sqrt{R_{ra}^2 - R_r^2} - t_b,$$

mod means remainder calculation, the letter $P_b$ is pitch of teeth, $R_{ra}$ denotes the inner radius of the ring gear teeth, $R_s$, $R_p$ denote the base radii of ring and planet gears, the angles $\alpha_1$ and $\alpha_2$ are pressure angles of the pitch circle of S-P and R-P meshes, respectively, and finally the arc length $t_b$ is the planet gear base circle tooth thickness. The mesh phasing difference with errors between S-P and R-P meshes $\Delta \psi_{sr}$ can be obtained as $\Delta \psi_{sr} = P_2 T / P_b$, which is shown in Figure 8. Therefore, when pin tangential error exists, gear mesh tooth phasing of S-P is $\Delta \psi_{sr}$ and R-P is $(\Delta \psi_{sr} + \Delta \psi_{rn})$.

**Numerical studies**

**Effect of pin flexibility on natural modes**

Natural frequencies are typically a major concern. To examine the effect of flexible pin on natural frequencies, the free vibration problem can be formulated by rewriting and casting the governing equation (5) into the normal mode form

$$\phi_i^T ([K_b + K_m] - \omega_i^2 M) \phi_i = 0$$

where vector $\phi_i$ is the eigenvalue vector, and value of $\omega_i$ is the corresponding eigenfrequency. In the present study, the bearing stiffnesses were calculated using a model based on the work of Lim and Singh. 23,24

Table 2 shows the vibration mode and natural frequencies of the case with 500 μm center distance error. There are 21 orders of natural frequencies that can be divided into 3 groups based on mode shapes. Six ranks of rotation modes, six ranks of translational modes (including 12 natural frequencies), and three ranks of planet modes. From Figure 6, the center distance error does have an effect on the mesh stiffness. When the
center distance error varies from 0 to 500 μm, the mean mesh stiffness change from $9.9626 \times 10^8$ to $9.5644 \times 10^8$ N/m (for external mesh) and $1.3010 \times 10^9$ to $1.3620 \times 10^9$ N/m (for internal mesh). But a slight change in gear mesh stiffness has little effect on the natural frequencies and this conclusion can also be drawn from Lin and Parker’s work.\(^1,2\) It may be noted that center distance error has almost no effect on natural frequencies.

In this study, to analyze the pin stiffness effect on natural frequencies, only the pin stiffness is varied, while assuming all others parameters invariant. Figure 9 shows the effect of pin stiffness on natural frequencies. In cases where that bearing stiffnesses is $10^8$ and $10^9$ N/m, only lower ranking frequencies are changed in trend when pin stiffnesses are more than $10^7$ N/m. However, high ranking frequencies are also obviously increased when pin stiffnesses are more than $10^9$ in cases that bearing stiffnesses are $10^{10}$ and $10^{11}$ N/m. Results showed pin stiffness obviously affects natural frequencies when the bearing stiffness is high. Also, the higher frequencies are more affected by the greater supporting stiffness values. Since, in general, the lower frequency range is more of interest in wind turbine transmission system, a suitably stiff planet pin stiffness is useful to avoid resonance because this substantially can reduce the number of natural frequencies as expected in the low frequencies range. However, the pin error has very little effect.

### Table 2. Natural frequencies (Hz) and corresponding modes of the wind turbine planetary gear train of interest with parameters in Table 1.

<table>
<thead>
<tr>
<th>Ranks</th>
<th>Frequencies</th>
<th>Modal shape</th>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>Sun, carrier, ring rotation</td>
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<tr>
<td>2</td>
<td>96</td>
<td>Sun, carrier, ring rotation</td>
</tr>
<tr>
<td>3, 4</td>
<td>204 (204)</td>
<td>Sun, carrier, ring translation</td>
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<tr>
<td>7</td>
<td>477</td>
<td>Planet motion</td>
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<td>8</td>
<td>485</td>
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<tr>
<td>9, 10</td>
<td>497 (497)</td>
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<td>21</td>
<td>1540</td>
<td>Sun, carrier, ring rotation</td>
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</table>

Figure 9. Variations of natural frequencies as pin stiffness increase with 500 μm center distance error and $k_b$ as the planet bearing stiffness. (a) $k_b = 3.36 \times 10^8$ N/m; (b) $k_b = 3.36 \times 10^9$ N/m; (c) $k_b = 3.36 \times 10^{10}$ N/m; (d) $k_b = 3.36 \times 10^{11}$ N/m.
Forced vibration analysis

In a gear system as shown in Figure 1, the effect of pin stiffness on vibration behaviors is analyzed next. When the wind turbine is working at its rated power, the input speed of the carrier is 17 r/min. For this condition, we assume the planet gear bearing stiffness to be $k_b = 3.36 \times 10^9$ N/m and the input torque to be $T_{in} = 9.33 \times 10^5$ Nm. Considering only a pin error $e_1$ on planet gear $p1$ and all the other planets are at their ideal position. And the mesh stiffness with pin error can be obtained from Figure 6. This is associated with pin position error as the external displacement excitation. Figure 10 shows the tooth mesh force on the gear surface in the case where pin stiffness is $2.13 \times 10^8$ N/m. It can be observed that the S-P mesh force is higher than the R-P mesh force. The tooth contact force deviation factors can be applied to observe the effect of different pin stiffness on the dynamic mesh.

The instantaneous forces on the tooth contacts are quantified by using the tooth contact force deviations defined as:

$$S_{rpm} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{N} \sum_{j=1}^{N} (F_{rpm}(ji) - F_m)^2 \right)^{\frac{1}{2}}$$

(11)

where values of $S_{rp}$ and $S_{rp}$ refers to S-P and R-P mesh tooth contact forces deviation factors, respectively, $n$ is the number of planets, $N$ means the number of time increments for averaging $S_{rp}$ and $S_{rpm}$, $F_{rpm}(ji)$ is instantaneous force between the sun gear and planet $j$, $F_{rpm}(ji)$ is instantaneous force between planet $j$ and the ring gear, and $F_m$ is mean force on one mesh (S-P or R-P). The effect of tooth contact forces deviation factors on changing pin stiffness with a floating gear configuration is shown in Figure 11. As shown in the Figure 11, the tooth contact force deviation factors will increase with increasing sun gear speed. But the tooth contact force deviation factor is less than 4000 N when sun gear speed is under 50 rad/s because of low available wind speed. It is also observed that the mesh load have larger variation at higher pin stiffness. Also, low spring constant flexible pins can decrease tooth contact force deviation factors.

Load sharing

In a 4-planet gear system, each S-P-R path is designed to transmit a quarter of the input torque. However, load sharing behavior can be associated with pin positional errors, which can cause one or more planets to lead or lag the other planets. In this case, pin stiffness and input load also play an important role in planet load sharing. In order to observe the effects of tangential and radial errors on load sharing in the case with the sun gear floating, the errors are prescribed on...
flexible pin 1 and all other flexible pins are assumed to be at their ideal position. Here, the sun gear ‘float’ is simulated by very low sun gear bearing stiffness. The operating conditions are the same as those used in Figure 10, pin error acts as the displacement excitation. Finally, the tangential supporting forces of the flexible pins are shown in Figure 12.

Several recent publications 4,11,12 have also shown the difference of load sharing behavior between non-floating systems and floating systems. In Figure 12, given a floating sun gear system, when there is only a pin tangential error $e_1$ on planet $p_1$ and all the other planets are without errors, $p_1$ and $p_3$ become more heavily loaded than $p_2$ and $p_4$. However, supporting loads of four planets have very little changed due to pin radial error. This conclusion correlates well with Singh’s finite element results,14 so this result detail is not discussed here. This reveals obvious influence of tangential error, but limited influence of radial errors. The pin radial error only causes a center distance change, and does not cause this pin to come in contact any sooner or later than the other pins. Hence, there is no direct influence on the mesh load sharing. This means that pin tolerance schemes should focus on minimizing the allowable pin tangential error.

Load sharing situation in the case of tangential error is shown in Figure 13. The results show that the load on planet 1 with error decreases linearly as error is reduced. Planet gear $p_1/p_3$ and $p_2/p_4$ carry almost equal loads in the case of the floating sun gear configuration. Here, the value of load percent $p_1$ is as large as 26% for $e_1 = 500 \mu$m. Meanwhile, for $e_1 = 500 \mu$m, planet gears $p_1/p_3$ carry higher loads than planet gears $p_2/p_4$.

In practical systems, every planet pin has error. Figure 14 defines the tangential error of the planet pins. Here, counter-clockwise is assumed to be the positive direction. Torque is transferred by the flexibility in the tangential direction, as the planet with deformation gets loaded. Here, a low sun gear bearing stiffness $2 \times 10^3$ N/m is applied to simulate floating sun gear.

In Figure 15, the maximum values of the load carried by the $n$-th planet gear with error as a percentage of the total input load are plotted for some common error configurations. Case C has the best percentage of load and case F is the worst. Case B is similar to case A except $e_1 = -e_2$, which suggest that the effective $e_1$ is now twice that used in case A. Case D with $e_2 = -e_1$ and case E with $e_1 = e_3$ have exactly the same results as case B, but the load percentages are higher than that of case B. It can also be noted that the case with error on opposite planets is more detrimental, from the view percentage of load carried, than that where the error is on adjacent planets. In case C with $e_1 = e_2$, and $e_3 = e_4 = 0$, the load sharing is in fact perfect. This means that the effect of pin error is less obvious on load sharing if the adjacent planet gear pins have the same error. Therefore, from the design viewpoint, it is
important to consider the pin tolerance of adjacent pairs of planets in the same direction. In the worst case F with \( e_1 = e_3 = -e_2 = -e_4 \), the maximum load increases to 32% for \( e_1 = 500 \mu m \) indicating that planet gears 2 and 4 each carry about 3.5% of the overload. This worst case correlates well with Bodas and Kahraman’s finite element results.\(^9\) In this case, the load percent is 32% when pins error is 500 \( \mu m \), while increased to 41% in case G where only pins stiffness is changed from flexible to rigid and others are same with case F. This worst load percent is also lesser than 38% of the Bodas and Kahraman’s rigid pins finite element result. This means flexible pin and floating sun gear system are more advantage to only floating sun gear on load sharing with pin error. Flexible pin may partly counteract distortion due to pin error.

In practical installations, since heavy load is usually transmitted by the wind turbine drives, the support stiffness and input torque will all play important roles on the deformation of planet gear support structures. Pin error, pin stiffness and input torque affect load sharing simultaneously. When tangential error exists, because of system flexibility and floating sun gear, the load caused by tangential error will be partly counteracted by loaded deflections and partly by the floating configuration. Also, load sharing will alter with changing input load. Therefore, to analyze this effect, the ratio \( Re \) is defined as the equivalent tangential error load to input load.

\[
Re = \frac{k_{ph} \times e}{F_{in}} = \frac{F_e}{F_{in}} = \frac{T_e}{T_{in}}
\]  

(12)

where, \( F_{in} \) and \( T_{in} \) are the total input force and torque, stiffness \( k_{ph} \) is total supporting stiffness of planet, and \( e \) is the pin tangential error, \( F_e \) and \( T_e \) are force and torque due by pin error. Here, system compliance only incorporates the pin stiffness and bearing stiffness because of lower flexible pin stiffness. The ratio \( Re \) considers the effects of the pin flexibility, the pin error, and the input load in the wind turbine planetary gear system. When load sharing is expressed as \( Re \) this is convenient for the analysis of support stiffnesses, error level and loading level in a planetary gear system with pin tangential error.

Figure 16 shows the load ratio values, i.e. the calculated load of the planet gear relative to the nominal load for each individual planet gear with changing \( Re \). As shown in the Figure 16, for a 4-planet gear system, the results show that the load ratio value can be transformed by changing the pin stiffness, pin error, or input load. Considering the results of Figure 13, this means all four planets are in contact with sun gear when \( Re < 1.0 \), whereas only two planets (p1, p3) are in contact when \( Re > 1.0 \). Also, negative errors have a similar load ratio to positive errors with the equivalent load increase in this 4-planet gear system. This indicates that lower pin stiffness, smaller pin tolerance and heavier input loads help to achieve better load sharing. Also, the load sharing condition will deteriorate with stiffer pin, greater pin tolerance and lighter loads cases.

**Conclusions**

In this study, the effects of flexible pins on the dynamic behavior of planetary gear system are investigated. The following specific conclusions are obtained

1. Increasing the center distance error of the pin tends to lower the actual contact ratio on external mesh, but on the other hand produces the opposite effect in the internal mesh.

2. Pin stiffness can affect natural frequencies significantly. When pin stiffness is increased, the number
of natural frequencies in the lower frequency range will be reduced as expected. However, pin error has little of this effect.

3. Lower pin stiffness can reduce the dynamic force induced by the spur gears shifting between one and two pairs of teeth in contact. At low sun gear speeds, the tooth contact force deviations of sun-planet and ring-planet mesh increase with increasing sun gear speed. Also, more compliant pins can decrease the tooth contact force deviation.

4. In a 4-planet system with floating sun gear, the pin tangential position error has a strong effect on load sharing. However, there is only limited influence from the pin radial error. In the tangential error case, load sharing depends heavily on the magnitude of the error, supporting stiffness, and input torque. Low pin stiffness, smaller pin tolerance and heavier loads will be better for load sharing. Adjusting pin stiffness can improve load sharing even when the pin tolerance values are large. This implies that the use of flexible pins may compensate for lower accuracy grade gears.

Funding
This research is supported by the Scholarship Award for Excellent Doctoral Student of China, SKLMT open funds of Chongqing University, Fundamental Research Funds for Central Universities of China (No. CDJXS10111135), the National Natural Science Foundation of China (No. 51175523), China National Key Technologies R&D Program (No. 2012BAA01B05).

Acknowledgments
The authors would like to thank anonymous reviewers for their helpful comments and suggestions to improve the manuscript. Authors also would like to the School of Dynamic Systems at the University of Cincinnati for their support of this research.

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Appendix

Notation

c_{sn}, c_{rn}  mesh damping of sun-planet gear and ring-planet gear

c_{pb}  supporting damping of planet gear

C  damping matrix of complete system

D  pitch circle diameter of planet gear

e  error value of flexible pin

f  force load applied to the flexible pin

F_{s, rpm}(j)  instantaneous force between sun-planet and ring-planet meshes

F_e, T_e  force and torque due to pin error

F_{in}, T_{in}  input force and input torque

F(t)  exciting force applied to express error excitation

G  gyroscopic matrix due to the rotation of planet gear

k_m  total mesh stiffness of internal or external pair gear

k_h  mesh stiffness due to Hertzian contact

k_{ib}, k_{is}, k_{ia}  mesh stiffness due to bending, shear and axial compression

k_{sn}(t), k_{rn}(t)  time-varying mesh stiffness functions with error of sun-planet gear and ring-planet gear

k_{i}, c_{i}  bearing supporting stiffness and damping of sun gear, ring gear and carrier (i = r, c, s)

k_{ui}, c_{ui}  circumferential stiffness and damping of sun gear, ring gear and carrier (i = r, c, s)

k_{pb}, k_{p}, k_{b}  planet gear supporting stiffness of total, pin and bearing

K_m, K_b  stiffness matrix associated with gear mesh and supporting structure

K_{ui}  stiffness matrix associated with the centrifugal effect of planet

K_{eff}  system stiffness

l  length of pin

m_i  masses of the carrier, ring gear, sun gear and planet gear (i = r, c, s, p)

M  mass matrix of complete system

N  number of time increments for averaging

R_e  load ratio of the tangential error load to the total input load

S_{s, rp}  deviation factors of the tooth contact forces of sun-planet and ring-planet meshes

t  time

t_n, r_n  tangential and radial displacements of planet gear

T  torque vector of input and output load of system

T_{in}  mean input torque

u_{i}  angular displacements of ring, carrier or sun gear (i = r, c, s)

\omega_c  input angular speed of carrier

x_{i}, y_{i}  translational displacements of ring, carrier or sun gear (i = r, c, s)

X  system displacement vector

Z_i  number of gear teeth

\alpha  pressure angle

\Delta_{ij}  compression displacement of elastic elements used in sun-planet-ring gear mesh and carrier-planet gear support mesh phasing difference

\Delta_{ij}  eigenvalue and eigenfrequency

\Psi_{n}  circumferential angle of n-th planet gear measured positive counterclockwise

Subscripts

c  carrier

n  number of planets

p  flexible pin of planet gear

r  ring

s  sun